# Nonlinearity Compensation based Tilting Controller for Electric Narrow Tilting Vehicles

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***Abstract*—Considering the traffic congestion and low energy consumption, small electric four-wheeled narrow tilting vehicles (NTV) are expected to be the new generation of city cars. In order to maintain lateral stability, the NTVs should have to lean into corners like two-wheeled vehicles. This is a challenge to keep a NTV stable during turning at different speeds. This paper aims to design a nonlinearity compensation based tilting controller for the direct tilting mechanism based NTVs. The controller adaptively compensates the nonlinearities of NTV roll dynamics in different vehicle speeds without the accurate vehicle models and, consequently, improve its robustness to rider's behaviour. By utilising the proposed nonlinear tilting control system, both new riders and experienced riders can drive the NTVs easily with improved tilting stability. Simulations have been conducted to validate the applicability and robustness of the proposed control approach.**

## I. Introduction

Due to the fact that the traffic congestion and parking problems influent people's daily life in urban area, small narrow commuter vehicles are recently being studied [1]–[4]. A narrow vehicle, which is a convergence of a car and a motorcycle, has four wheels but just half the width of a conventional car. This kind of vehicles are called narrow tilting vehicles (NTV) because they have to lean into corners like a two-wheeled vehicles [5], [6], as shown in Fig. 1. Considering the practical dimensions and low energy consumption, the NTVs are expected to be the new generation of city cars. However, their narrow width and high center of gravity make the roll stability of NTVs a challenge. Unlike the case of a two-wheeled vehicle that the driver tilts the vehicle by his own weight, the tilting of an NTV should be automatically acted by a mechanical system with a tilting controller.

NTV tilting control can be mainly classified into two mechanical systems: one is the steering tilt control (STC), which controls directly on the steering angle of front wheel to stabilize the tilt mode of the vehicle [7], [8]; the other is the direct tilt control (DTC), which provides addition moment of torque to tilt the vehicle into a desierd corner, as shown in Fig. 2. In previous studies, although the STC system is efficient at high speed, it shows a major disadvantage that the balancing does not suit well at the standstill or very low speeds. In addition, in slippery road conditions, the lack of tyre friction makes it difficult to balance the vehicle in a safe situation [9].

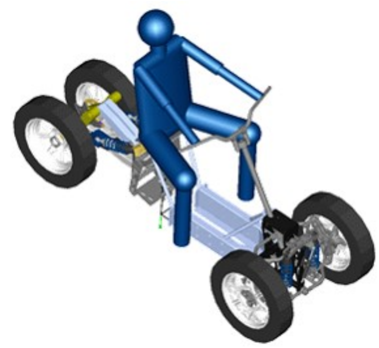

Fig. 1. The diagram of narrow tilting vehicle in RESOLVE project developed by AIT.

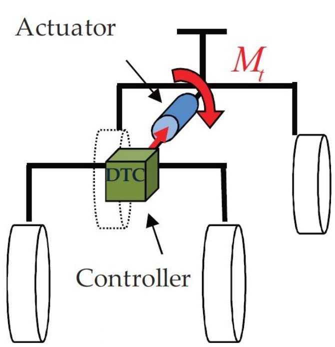


Fig. 2. The DTC tilting mechanisms of NTV by [3].

The DTC based mechanism slightly simplifies these control problems with a seperate tilt actuator as an additional control input to lean the vehicle to a desired tilt anlge [10]. The main challenge of the DTC system is that it requires high tilting motion at high vechile speed and the delayed vehicle response and the risk of vehicle oscillations, thus requiring highly sophisticated loop control algorithms, which also need to be adaptive to different loads and driving conditions [10].

Althoug the dual mode switching strategies that involve the STC and DTC systems working together to reduce the drawbacks of both mechanisms, such approaches have obvious discontinuous behaviour [7], [10]–[12]. Besides of these, several studies focused on using different control strategies to improve the performance of DTC mechanisms. Linear SISO control approaches have been used to drive the DTC-based actuators [13], [14]. But such linear control approaches cannot perform a natural decoupling between the longitudinal and the lateral dynamics consequently and achieve only the desired performance at low vehicle speeds. Considering the coupling dynamics of NTVs, some model-based control methods are employed to optimise the vehicle response in lateral dynamics [3], [15]–[17]. These control methods just consider the steady state vehicle behaviour, and the non-linear dynamics have been ignored. In addition, some nonlinear control solutions are used to compensate the nonlinear behaviour of NTVs, such as gain-scheduling and feedback linearization methods [16], [18]. However, these existing control solutions require the knowledge of an accurate vehicle model to design the pre-scheduled controller gains or linearize the vehicle model.

This research, therefore, aims to design a non-linear tilting controller for DTC-based NTVs. This approach does not require an accurate vehicle model but capable of compensating the nonlinearities of vehicle tilting dynamic in different vehicle speeds in order to improve the controller robustness of longitudinal vehicle variations and rider's behaviour. Thus, by using this control system, both new and experienced riders can ride the NTVs easily with improved tilting stability. Simulations on a nonlinear model of NTV have been carried out to validate the performance of the proposed nonlinear tilting control by comparing to the conventional linear and gain-scheduling control methods.

## II. System Modelling

### A. NTV System Description

There are several models of NTV have been proposed in literatures [1], [7], [11], [19]. University of Bath has proposed a five DoF nonlinear model from the "CLEVER" project [5], [12], [15]. The University of Minnesota has proposed several nonlinear models in both linear and nonlinear [10], [16], [20]. The front wheels and rear wheels are represented by single wheels in a basic 'bicycle' model to reduce the complexity. This model consideres the longitudinal and lateral motion of vehicle dyanmics but not the pitch motion. One difference between a bicycle and a NTV system is how the riders can tilt them. In balancing a lightweight bicycle in turn, the rider can shift his weight to lean the bicycle into a corner. But in riding a NTV, the mass of a rider is much less than that of a vehicle. The rider has to act on the steering and throttle to balance the vehicle [5]. Therefore, in modelling a NTV system, the vehicle and its rider (and passenger) can be seen as a single mass model, as seen in Fig. 3.

In addition, the lateral dynamics of vehicle are a result of the friction force on tyre and the generated moment through road-tyre interaction. A slip-dependent tyre force model is introduced on behave of the real tyre behaviour at a ground-contact point. When the sideslip increases, there is more chance of sliding at rear wheels reply on the road conditions. Once all the road-contact elements begin to slide, the force saturation is reached and the vehicle cannot be maintained stable.

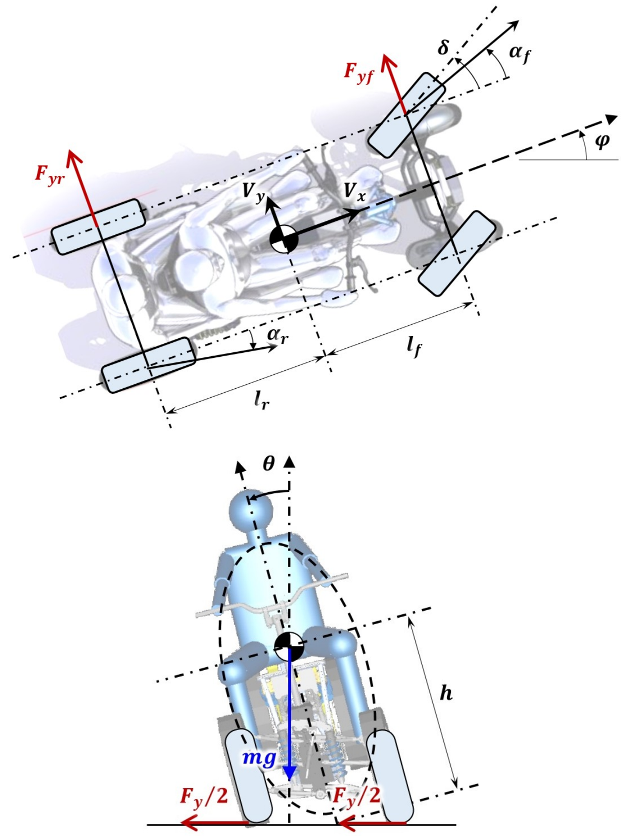


Fig. 3. Geometry of a turning vehicle.

The role of rider can be seen as an active controller [21], which validates the influence of rider's mottions on the vehicle behaviour. In that research, the rider is treated as a feedback compensator between roll angle and steering torque in bicycles. But in DTC-based NTVs, the tiltling mechanism is desired to control the vehicle roll angle and rider is only to track the yaw rate by steering and longitudinal speed by throttle.

### B. Mathematical Model of Vehicle Roll and Lateral Dynamics

The nonlinear model of the roll lateral dynamics of NTV is shown as:

$$\ddot{\theta} = \tfrac{1}{I_x + mh^2\sin^2\theta} \quad \begin{bmatrix} mhg\sin\theta - mh^2\dot{\theta}^2\sin\theta\cos\theta \\ -h\cos\theta(F_{yf}+F_{yr}) + M_t \end{bmatrix} \tag{1}$$

$$\begin{aligned} F_{yf} &= \quad 2C_f\alpha_f + 2\lambda_f\theta \\ F_{yr} &= \quad 2C_r\alpha_r + 2\lambda_r\theta \end{aligned} \tag{2}$$

$$\begin{aligned} \alpha_f &= \quad \delta - \tan^{-1}\left(\tfrac{l_f\dot{\varphi}+V_y}{V_x}\right) \\ \alpha_r &= \quad -\tan^{-1}\left(\tfrac{-l_r\dot{\varphi}+V_y}{V_x}\right) \end{aligned} \tag{3}$$

$$\dot{V}_y = \frac{1}{m}\left(F_{yf}+F_{yr}\right) - V_x\dot{\varphi} - h\ddot{\theta}\cos\theta + h\dot{\theta}^2\sin\theta \tag{4}$$

$$\ddot{\varphi} = \frac{1}{I_z}\left(l_f F_{yf} - l_r F_{yr}\right) \tag{5}$$

where $\theta$ and $\varphi$ are the roll angle and yaw angle of vehicle. $\delta$ is the steering angle of front wheel; $M_t$ is the moment of torque in tilting. $m$ is the vehicle mass, $h$ is the height of center of gravity (COG), $g$ is the gravitational constant. $V_x$ and $V_y$ are the longitudinal and lateral speed of vehicle. $F_{yf}$ and $F_{yr}$ are the lateral tyre force of front and rear wheels, respectively. $I_x$ and $I_z$ are the vehicle roll and yaw moment of inertia, respectively. $C_f$ and $C_r$ are the front and rear cornering stiffness; $\lambda_f$ and $\lambda_r$ are the front and rear camber stiffness; $\alpha_f$ and $\alpha_r$ are the front and rear tyre side-slip angle. $l_f$ and $l_r$ are the distance from COG to front and rear axle.

The model can be simplified for controller design by assuming $\dot{V}_y = 0$ and $\alpha_f = \alpha_r$. The simplified vehicle tilt angle dynamic and yaw angle dynamic are shown as:

$$\ddot{\theta} = \frac{1}{I_x + mh^2}\left(mhg\sin\theta - mhV_x\dot{\varphi}\cos\theta + M_t\right) \tag{6}$$

$$\dot{\varphi} = \frac{V_x \tan\delta}{l_f + l_r} \tag{7}$$

Then the simplified model can be linearized depend on the longitudinal vehicle speed. The linearized model can be represented as:

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix}\theta \\ \dot{\theta} \\ \varphi\end{bmatrix} = & \begin{bmatrix} 0 & 1 & 0 \\ \frac{mhg}{I_x + mh^2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix}\theta \\ \dot{\theta} \\ \varphi\end{bmatrix} \\ & + \begin{bmatrix} 0 & 0 \\ \frac{mhV_x^2}{(I_x + mh^2)(l_f + l_r)} & \frac{1}{I_x + mh^2} \\ \frac{V_x}{l_f + l_r} & 0 \end{bmatrix}\begin{bmatrix}\delta \\ M_t\end{bmatrix} \end{aligned} \tag{8}$$

The linearized model is used for the design of linear tilting controller, the PI and gain-scheduling PI controller.

For the vehicle route tracking, the route path is defined as a function of longitudinal speed and yaw rate as:

$$\begin{aligned} \dot{x} &= V_x\cos\varphi \\ \dot{y} &= V_x\sin\varphi \end{aligned} \tag{9}$$

Thus, the virtual rider is designed using a simple PI controller as

$$\delta = \left(K_p + \frac{K_i}{s}\right)\left(\varphi^* - \varphi\right) \tag{10}$$

where $\varphi^*$ is the yaw angle reference to track the route path.

## III. Nonlinear Tilting Controller Design

In the nonlinear controller design, the first step is to linearize the relation between system input and output, which is called input-output linearization. The tilting angle quation can be rewritten as

$$\ddot{\theta} = L_f(x) + B(\theta)M_t \tag{11}$$

where

$$\begin{aligned} L_f(x) = & \frac{1}{I_x + mh^2\sin^2\theta}\Big[mhg\sin\theta - mh^2\dot{\theta}^2\sin\theta\cos\theta \\ & -h\cos\theta\left(2C_f\delta - 2C_f\tan^{-1}\left(\frac{l_f\dot{\varphi} + V_y}{V_x}\right)\right. \\ & \left.\left. +2\lambda_f\theta + 2C_r\tan^{-1}\left(\frac{-l_r\dot{\varphi} + V_y}{V_x}\right) + 2\lambda_r\theta\right)\right] \end{aligned} \tag{12}$$

$$B(\theta) = \frac{1}{I_x + mh^2\sin^2\theta} \tag{13}$$

As $B(x) \neq 0$ if $I_x > 0$, the $B(x)$ is non-singular for all nominal operating points. The system control input can be designed as

$$M_t = B^{-1}\left(-L_f(x) + u_t\right) \tag{14}$$

where $u_t$ is the control input to the linearized system and it can be designed as

$$u_t = k_1\left(\theta^* - \theta\right) - k_2\dot{\theta} \tag{15}$$

and $\theta^*$ is the ideal tilt angle calculated as a function of steering angle and vehicle speed as

$$\theta^* = \tan^{-1}\left(\frac{V^2\delta}{(l_f + l_r)g}\right) \tag{16}$$

Choose $B_0 = B(\theta)\|_{\theta=0}$ as the nominal control gain at rated value. The difference between $B(x)$ and $B_0$ will be seen as disturbance in the lumped perturbation. Define the perturbation terms as

$$\ddot{\theta} = \Psi(x) + B_0 M_t \tag{17}$$

where

$$\begin{aligned} \Psi(x) &= L_f(x) + (B(x) - B_0)M_t \\ B_0 &= \frac{1}{I_x} \end{aligned} \tag{18}$$

Assume the lumped perturbation will not change quicker than one time cycle. Calculate the perturbation term as

$$\hat{\Psi}(t) \approx \Psi(t-1) = \ddot{\theta}(t-1) - B_0 M_t(t-1) \tag{19}$$

The final control input is

$$M_t = B_0^{-1}\left[-\hat{\Psi} + k_1\tan^{-1}\left(\frac{V_x^2\delta/g}{l_f + l_r}\right) - k_1\theta - k_2\dot{\theta}\right] \tag{20}$$

Fig. 4 shows the block diagram of the nonlinear control for tilting mechanism of NTV.

## IV. Simulation Result

The NTV parameters used for the simulation are from [22] given in Table I. The controllers used in simulation are with the parameters given in Table II. The simulation validations are carried out with tracking the route of shape '$\infty$'. In the route profile, the speed reference is kept constant and the yaw rate reference is in a square wave.

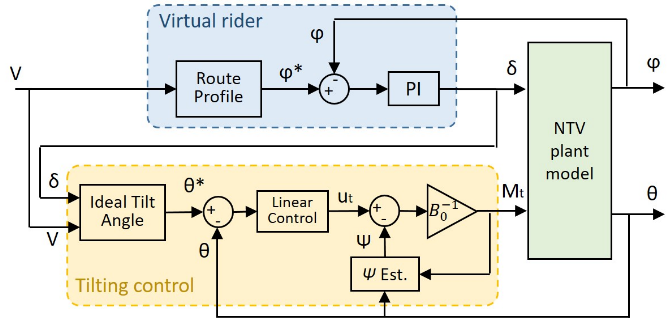


Fig. 4. Control block diagram of the tilting control for the tilting mechanism of NTV.

TABLE I
SYSTEM PARAMETERS OF NTV

| Description | Symbol | Value | Unit |
|---|---|---|---|
| Total vehicle mass | $m$ | 96.0 | kg |
| Height of vehicle COG | $h$ | 0.25 | m |
| Gravitational constant | $g$ | 9.81 | $\mathrm{m/s^2}$ |
| Distance from COG to front axle | $l_\mathrm{f}$ | 0.69 | m |
| Distance from COG to rear axle | $l_\mathrm{r}$ | 0.84 | m |
| Vehicle roll moment inertia | $I_\mathrm{x}$ | 18 | $\mathrm{kg{\cdot}m^2}$ |
| Vehicle yaw moment inertia | $I_\mathrm{z}$ | 60 | $\mathrm{kg{\cdot}m^2}$ |
| Front cornering stiffness | $C_\mathrm{f}$ | 3500 | N/rad |
| Rear cornering stiffness | $C_\mathrm{r}$ | 5480 | N/rad |
| Front camber stiffness | $\lambda_\mathrm{f}$ | 1000 | N/rad |
| Rear camber stiffness | $\lambda_\mathrm{r}$ | 2000 | N/rad |

TABLE II
CONTROL PARAMETERS

| | |
|---|---|
| Virtual rider | $K_\mathrm{p} = 0.1$<br>$K_\mathrm{i} = 0.1$ |
| Nonlinear tilting controller | $k_1 = 300$<br>$k_2 = 400$<br>$B_0 = 0.0556$ |

### A. Constant Speed Tilting Test

Fig. 5 shows the simulation result of the vehicle yaw rate and tilting angle performance at the constant speed of 20 km/h. The virtual rider controls the vehicle yaw rate with the predefined reference of route path to be tracked. Under the step changed yaw rate reference, the vehicle yaw rate can track the reference well. The vehicle tilting angle is controlled by the tilting mechanism with the proposed nonlinear controller. The tilting angle is smooth and track the desired tilting angle to keep the vehicle stable during the turning.

In order to verifiy the robustness of the proposed controller against the noise and disturbance, random noise was added in the feedback signal to simulate the controller working under real sensor noise. The result in Fig. 5 is comparing between the control performance with ideal signals and signals with feedback noise. From the result one can find that the controller performance will not have major effect with the noise even though the feedback linearization method has been used in the controller design. This is to verify that the controller is robust to unknown noise in operation.

The route path tracking performance is shown in Fig. 6, where the desired path is in black line and the vehicle path is in the blue line. The vehicle path track is due to the yaw rate control of a virtual rider and the stability is controlled by the tilting mechanism. The entire control performance is good that the vehicle tracks the path well.

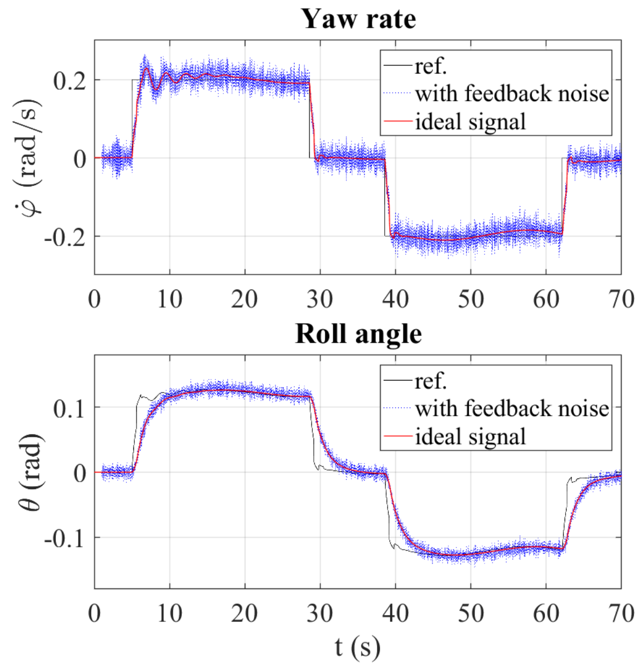


Fig. 5. Simulation result of vehicle yaw rate and tilting angle at constant speed of 20km/h.

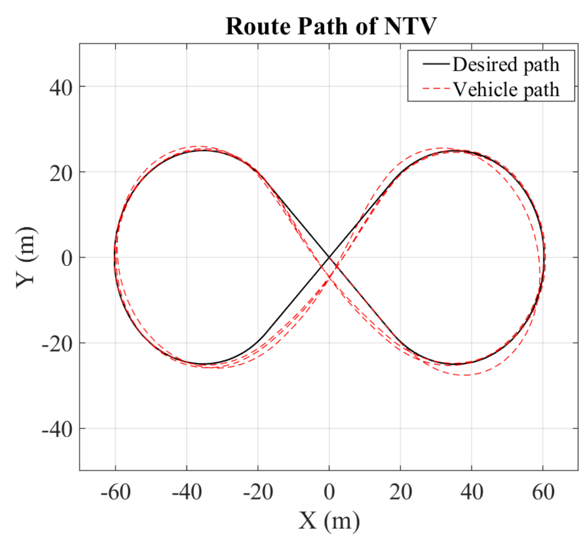


Fig. 6. Simulation result of vehicle route path tracking performance.

### B. Increasing and Decreasing Speed Tilting Test

In the following sections, the same route path is used for the validation but the speed is changed from low speed to high speed. With the change of vehicle speed, the desired yaw rate and tilting angle also amended depend on the speed. The dynamics of the NTV under different speed and tilting angle has nonlinear behaviour. The linear PI controller based tilting control, the gain-scheduling PI based tilting control, and the nonlinearity compensation based tilting control has been validated and compared among their control performance. Fig.

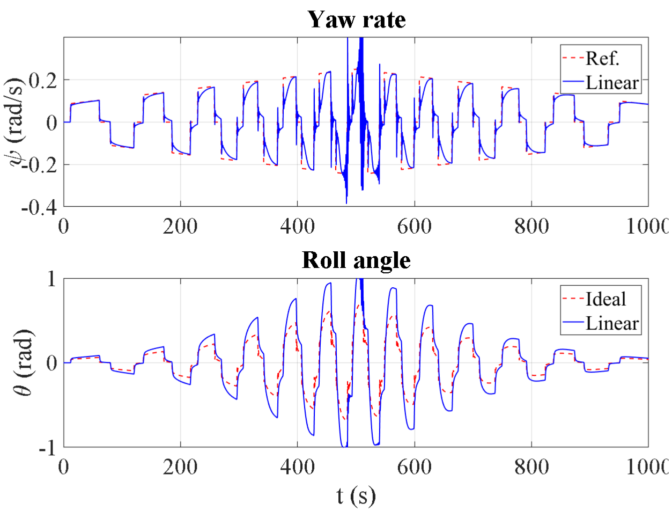


Fig. 7. Simulation result of yaw rate and tilt angle under the speed from 5 km/h to 45 km/h with Linear Controller.

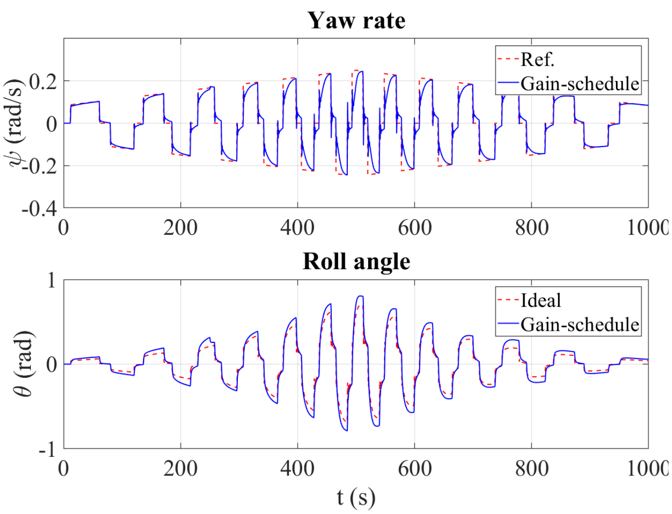


Fig. 8. Simulation result of yaw rate and tilt angle under the speed from 5 km/h to 45 km/h with Gain-scheduling Controller.

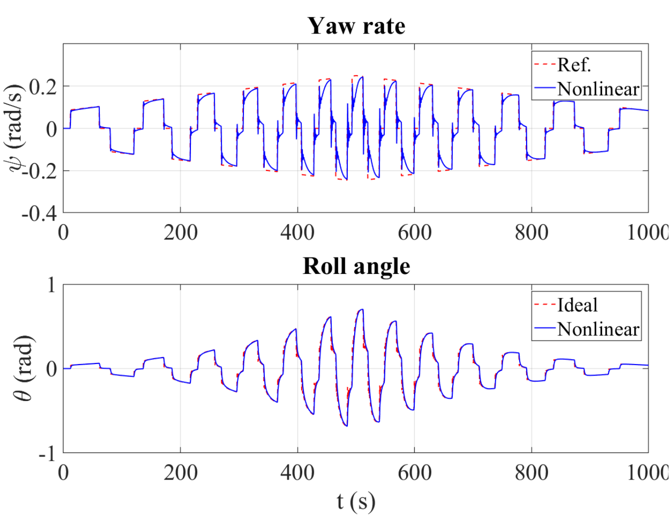


Fig. 9. Simulation result of yaw rate and tilt angle under the speed from 5 km/h to 45 km/h Nonlinear Controller.

7 to 9 show the simulation result of the vehicle yaw rate and tilting angle under the speed from 5 km/h to 45 km/h with the tilting controller of linear one, gain-scheduling one, and the nonlinear one.

From the results above, it is obviously to find that the linear controller cannot perform well enough at high vehicle speed. That is because the linear controller is designed based on the linear model that is linearized at a target speed. When the real vehicle speed is beyond the speed linearization region, the linear controller cannot be guaranteed to perform well in controlling the system.

The gain-scheduling (GS) controller is to separate the non-linear control region to small regions, and in each small region the linear control law is used, and switch the controller among the regions. In this paper, the GS controller schedules the gains in three regions. In each region, the controller performs well enough in the tilting control. However, when switching between the controllers, the control performance is reduced as additional disturbance includes.

The nonlinear controller is designed to cover the whole operating region from low speed to high speed. And the nonlinearity caused by speed changes has been compensated with the calculated perturbation. This is to optimize the control performance of the tilting controller in the entire operating region. The nonlinear behaviour of vehicle is considered in the nonlinear controller design, and the nonlinear controller has been validated with the best performance comparing to the linear controller and GS controller with less tracking error.

Fig. 10 shows the performance comparison among different controllers rely on the integral absolute error (IAE) of yaw rate and roll angle. All the three controller are applied on the same system with the same vehicle and virtual rider models. The roll angle tracking IAE of the nonlinear controller is 46% less than that of the GS controller and 75% less than that of the linear controller. In addition, through all control systems are using the same virtual rider model for yaw rate tracking, the performances are different in the results. The better performance the tilting controller actied in roll angle trakcing, the easier its rider can track the reference yaw rate. In the nonlinear controller based system, it tracks the target yaw rate with 24% less IAE of the linear one and 9% less IAE of the GS one. These results show that the nonlinear tilting controller not only performs better in roll angle tracking, but also can help the rider in yaw rate tracking and route path following.

## V. Conclusion

This paper has proposed a nonlinear tilting controller to improve the stability of a direct tilting mechanism based electric narrow tilting vehicle. The approach has the capability to compensate the nonlinearities of vehicle tilting dynamics without the requirement of an accurate vehicle model. The designed nonlinear tilting controller has been validated in simulation on a vehicle and virutal rider system. The results show that nonlinear controller performs better than the con-ventional linear and gain-scheduling controllers with less roll

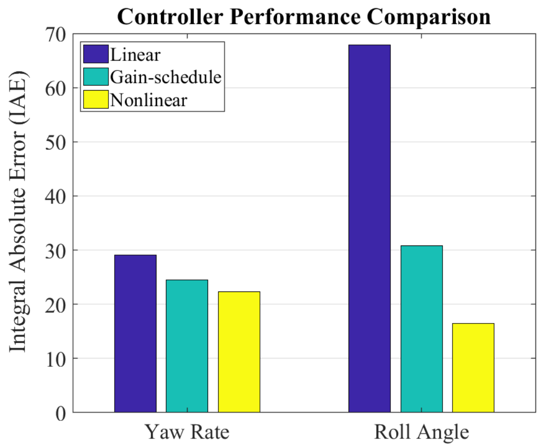


Fig. 10. Controller performance comparison among their IAE of yaw rate and tilt angle.

angle tracking error under different speeds. In addition, it can help rider to improve the yaw rate tracking performance. With the improved robustness to longitudinal speed and rider's behaviour, both new and experienced riders can rider the NTVs easily by using this tilting controller.

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